Quiz 4C, Business Calculus

Fall 2012

3:23
3:30
7
⇒ 20 minutes.

Name: Key

1. (4 points) Let $f(x) = -x^3 + 3x^2 + 24x - 10$. Find the following, and make sure to show your work.
   (a) The relative minimum of f, if one exists. Write your answer as (x, y) coordinates.
   (b) Find the interval where f is concave up.
   (c) Find the inflection point(s) of f, if they exist.
   (d) Find where f is increasing.

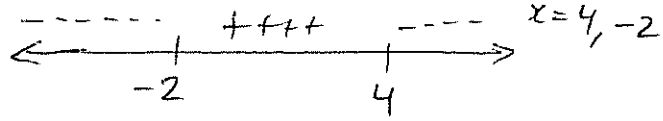
$f'(x) = -3x^2 + 6x + 24$

$f''(x) = -6x + 6$

$f'(x) = 0 \Rightarrow 0 = -3x^2 + 6x + 24$

$0 = -3(x^2 - 2x - 8)$ ✓

$0 = -3(x-4)(x+2)$



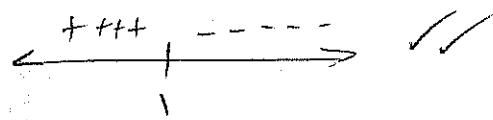
(a) relative min at $x = -2$

$\Rightarrow (-2, -38)$ ✓

$f(-2) = -(-2)^3 + 3(-2)^2 + 24(-2) - 10$

$= 8 + 12 - 48 - 10 = -38$

(b) $-6x + 6 = 0 \Rightarrow x = 1$



✓✓

concave up on $(-\infty, 1)$ ✓

(c) inflection point $= (1, 16)$ ✓

$-1 + 3 + 24 - 10 = 16$

(d) increasing on $(-2, 4)$ ✓

2. (4 points) The total cost incurred in operating a certain kind of truck, traveling at a speed of v mph, is estimated to be

$$(C(v) = 125 + v + \frac{4500}{v}$$

where C is measured in dollars. Use differentials to estimate the change in total cost when the average speed is increased from 50 to 52 mph.

$C'(v) = 1 - 4500\,v^{-2} = 1 - \frac{4500}{v^2}$ ✓✓

$dC = C'(v_0)\,dv$ ✓✓

$v_0 = 50$ ✓

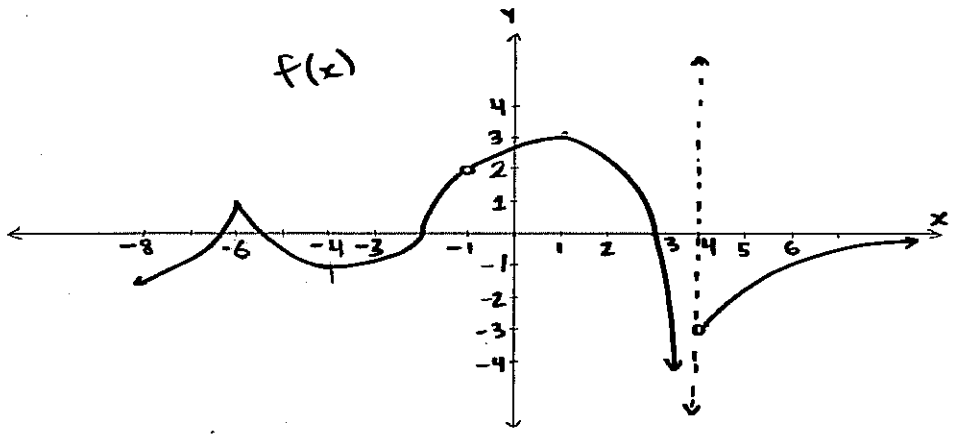
$dv = 2$ ✓

$C'(v_0) = 1 - \frac{4500}{2500} = 1 - \frac{9}{5} = \frac{-4}{5}$ ✓

$dC = \frac{-4}{5}(2) = \frac{-8}{5} = -1.60$

$\Rightarrow$ total cost decrease by \$1.60

3. (2 points) Let the following graph represent $f(x)$. Answer the questions below.



(a) $f(x)$ is decreasing on the interval(s): $(-6,-4)$, $(1,4)$ ✓

(b) $f(x)$ is concave down on the interval(s): $(-2,-1)(-1,4)$, $(4,\infty)$ ✓✓

(c) $f(x)$ has local maximum(s) at x-values of: -6, 1 ✓

Quiz 4D, Business Calculus

Fall 2012

Name: Key

1. (4 points) Let $f(x) = -x^3 + 3x^2 + 24x - 10$. Find the following, and make sure to show your work.

   (a) The relative maximum of f, if one exists. Write your answer as (x, y) coordinates.

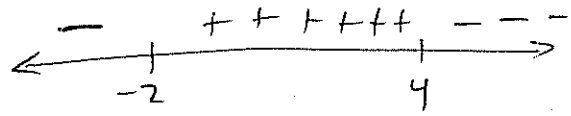
   (b) Find the interval where f is concave down.

   (c) Find the inflection point(s) of f, if they exist.

   (d) Find where f is decreasing.

✓ $f'(x) = -3x^2 + 6x + 24 = -3(x^2 - 2x - 8) = -3(x-4)(x+2)$

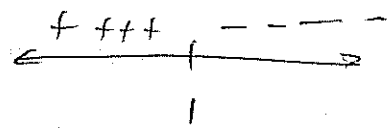
$f'(x) = 0$ at $x = 4, x = -2$



$-64 + 48 + 96 - 10 = 70$

✓ (a) max at $x = 4 \Rightarrow$ $\boxed{(4, 70)}$

✓✓ (b) $f''(x) = -6x + 6 = -6(x-1)$



$\Rightarrow$ concave down on $\boxed{(1, \infty)}$ ✓

$-1 + 3 + 24 - 10$ ✓

(c) Inflection point at $x = 1 \Rightarrow (1, f(1)) = \boxed{(1, 16)}$ ✓

(d) decreasing where $f'(x) < 0$

$\Rightarrow$ $\boxed{(-\infty, -2) \text{ and } (4, \infty)}$ ✓

2. (4 points) The total cost incurred in operating a certain kind of truck, traveling at a speed of v mph, is estimated to be

$$C(v) = 125 + v + \frac{4500}{v}$$

where C is measured in dollars. Use differentials to estimate the change in total cost when the average speed is decreased from 55 to 52 mph.

$C'(v) = 1 - \frac{4500}{v^2}$ ✓✓

$dC = C'(v_0)\,dv$ ✓✓

$v_0 = 55$ ✓

$dv = -3$ ✓

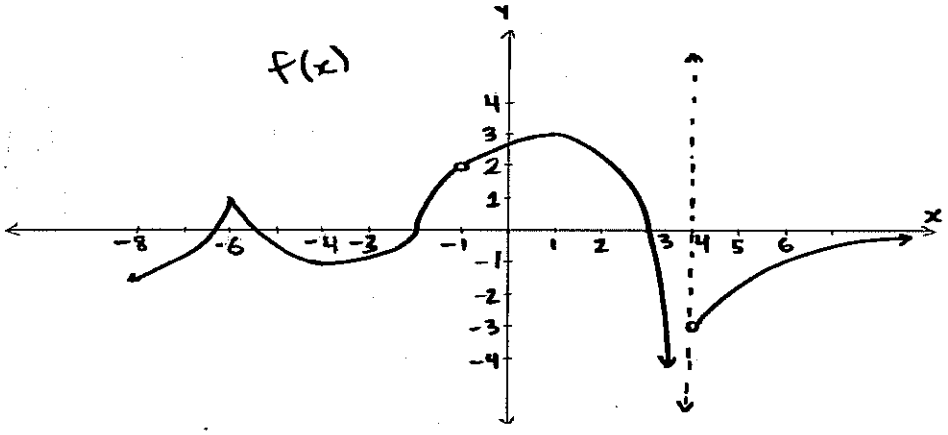
$dC = C'(55) \cdot (-3)$ ✓

$= \left(1 - \frac{4500}{55^2}\right)(-3)$ ✓

$= \left(1 - \frac{4500}{3025}\right)(-3) \approx 1.5$

$\Rightarrow$ \$1.50

3. (2 points) Let the following graph represent $f(x)$. Answer the questions below.



(a) $f(x)$ is increasing on the interval(s): $(-\infty, -6), (-4, -1), (-1, 1), (4, \infty)$ ✓✓

(b) $f(x)$ is concave up on the interval(s): $(-\infty, -6), (-6, -2)$ ✓

(c) $f(x)$ has inflection point(s) at $(x, y) =$ $(-2, 0)$ ✓